



Lecture 14 Optical implementation of Deutsch algorithm

(1)

- Optical implementation of Deutsch algorithm
- Special Topic: Quantum Network

optical two-qubit state $|x\rangle|y\rangle$

$|0\rangle|0\rangle$

path polarization

$|x\rangle|y\rangle$

two-rail construction

$|V\rangle|0\rangle$

$|H\rangle|1\rangle$

(2)

$|\phi\rangle = \cos\phi |V\rangle + \sin\phi |H\rangle$
 $= \cos(\frac{\theta}{2}) |V\rangle + \sin(\frac{\theta}{2}) |H\rangle$

$2\theta = 90^\circ$
 $\theta = 45^\circ$

optical network

HWP

HWP (45°)

X gate

$|V\rangle \rightarrow |H\rangle$
 $|H\rangle \rightarrow |V\rangle$

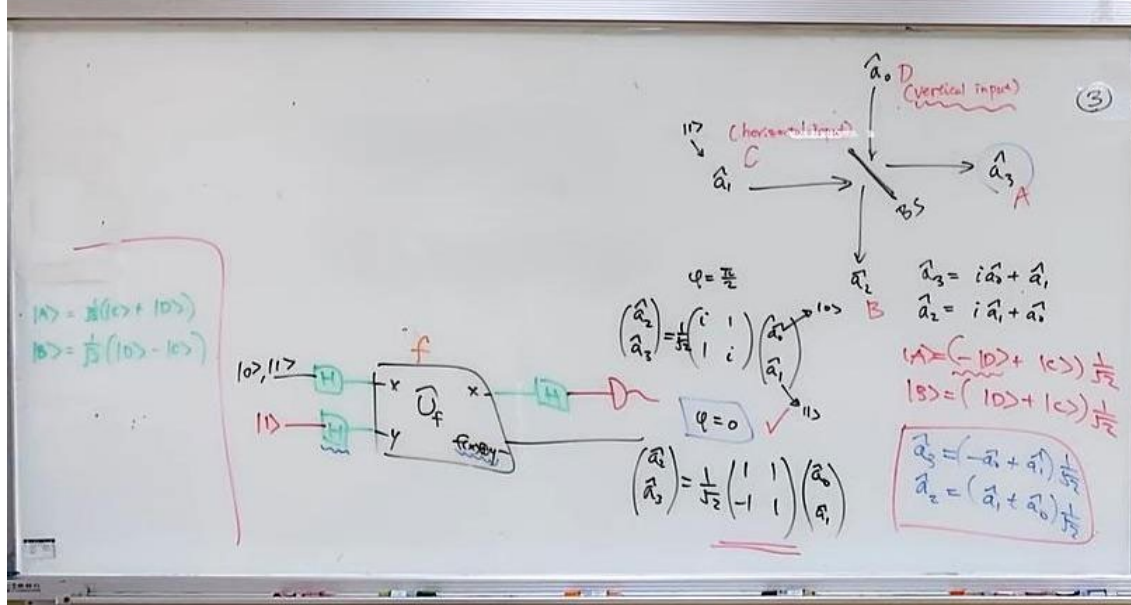
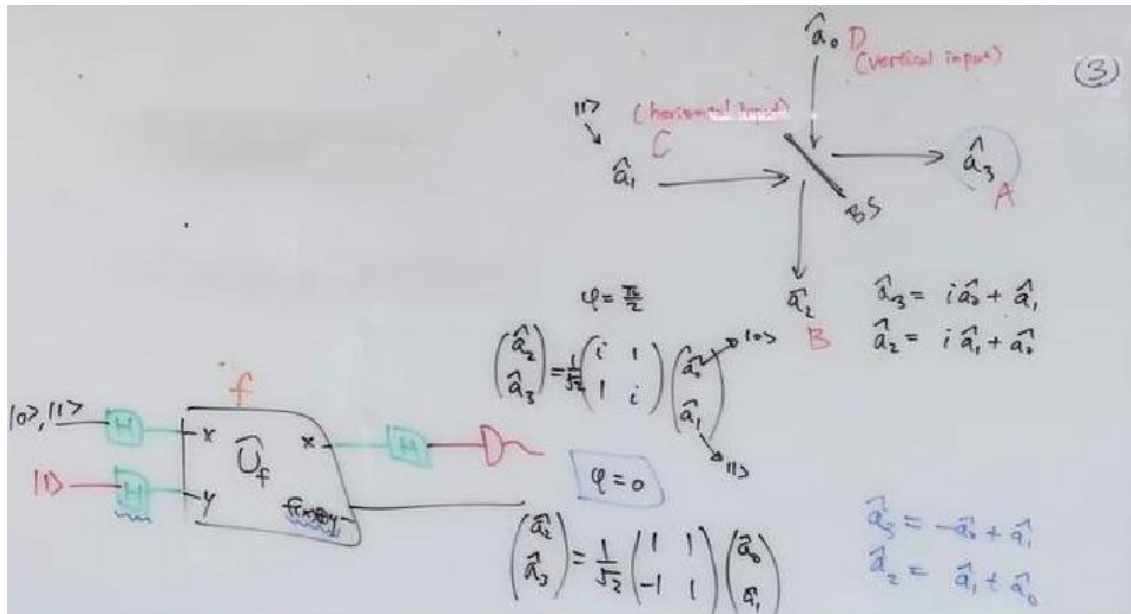
$1/2$ Hadamard gate

$|V\rangle \rightarrow \frac{1}{\sqrt{2}}(|V\rangle + |H\rangle)$
 $|H\rangle \rightarrow \frac{1}{\sqrt{2}}(|V\rangle - |H\rangle)$

$|\phi_0\rangle = \frac{1}{\sqrt{2}}(|V\rangle + |H\rangle)$
 $|\phi_H\rangle = \frac{1}{\sqrt{2}}(|V\rangle - |H\rangle)$

$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} V \\ H \end{pmatrix}$







④

$|A\rangle = |C\rangle |D\rangle$
 $|B\rangle = |C\rangle |D\rangle$

$B = a_2 = ia_0 - a_1 = iD - C$
 $A = a_3 = -a_0 + ia_1 = -D + iC$

$|A\rangle = \frac{1}{\sqrt{2}}(|D\rangle + |C\rangle)$
 $|B\rangle = \frac{1}{\sqrt{2}}(|C\rangle + |D\rangle)$

$|B\rangle = \frac{1}{\sqrt{2}}(e^{i\pi/2}|C\rangle + |D\rangle)$
 $|A\rangle = \frac{1}{\sqrt{2}}(e^{i\pi/2}|D\rangle + |C\rangle)$

④

$|A\rangle = \frac{1}{\sqrt{2}}(|D\rangle + |C\rangle)$
 $|B\rangle = \frac{1}{\sqrt{2}}(|D\rangle + |C\rangle)$

$\begin{pmatrix} B \\ A \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix}$

$\begin{pmatrix} B' \\ A' \end{pmatrix} = \begin{pmatrix} B \\ -A \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix}$

$\begin{pmatrix} a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \sin\theta & e^{i\phi}\cos\theta \\ -e^{i\phi}\cos\theta & \sin\theta \end{pmatrix} \begin{pmatrix} a_1 \\ a_0 \end{pmatrix}$



The image contains two handwritten quantum circuit diagrams. The left diagram shows a circuit with two input qubits, a Hadamard gate, a unitary U_f , and a measurement. The right diagram shows a more complex circuit with multiple qubits, Hadamard gates, a unitary U_f , and a measurement. It includes handwritten calculations for the probability of outcome 1.

Left Diagram:

- Two input qubits are shown, labeled $|0\rangle$ and $|1\rangle$.
- The top qubit passes through a Hadamard gate (H) and then a unitary U_f .
- The bottom qubit passes through a Hadamard gate (H) and then a unitary U_f .
- The outputs of both qubits are measured.

Right Diagram:

- Two input qubits are shown, labeled $|0\rangle$ and $|1\rangle$.
- The top qubit passes through a Hadamard gate (H) and then a unitary U_f .
- The bottom qubit passes through a Hadamard gate (H) and then a unitary U_f .
- The outputs of both qubits are measured.
- Handwritten calculations for the probability of outcome 1 are shown:
$$110 \equiv |1\rangle|0\rangle$$

$$100 \equiv |0\rangle|1\rangle$$

$$110 \equiv |0\rangle|1\rangle$$

$$100 \equiv |0\rangle|1\rangle$$

The calculations show that the probability of outcome 1 is 1, and the probability of outcome 0 is 0.